Geodesy 1B (GED209)
Lecture No: 9

Reduction To Center

Ali ELSAGHEER, Mohamed FREESHAH, Reda FEKRY
reda.Abdelkawy@feng.bu.edu.eg

Problem Definition

- In a triangulation network, mosques, church spires, or any similar tall objects are marked as triangulation stations.
- Such types of stations cannot be occupied.
- Consequently, the instrument is set up on an auxiliary station which is so close to the main station.
- This auxiliary station is called "**Satellite Station**"
- At the satellite station, all angles to the adjacent stations are measured with the same precision of other angles in the system.
- The process of computing the main angle at the inaccessible station from the measured ones is called "**Reduction to center**"

Problem Definition

GED Geomatics Engineering Department

- From the shown figure:

Y, T, and ***B***: triangulation stations

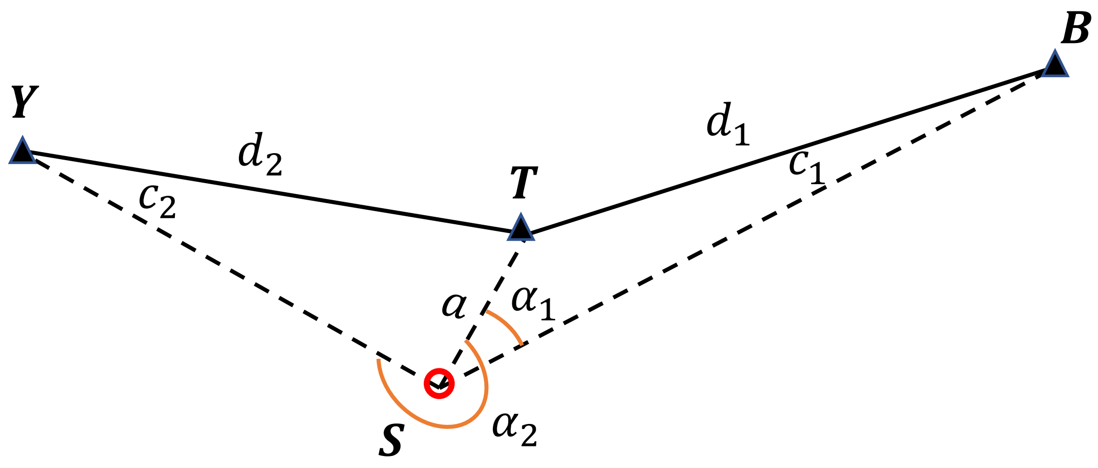
T: inaccessible triangulation station

S: satellite station

α_1 , α_2: observed angles

a : distance between satellite station and inaccessible station.

By solving triangles TSB, and TSY



$$\frac{\sin c_1}{a} = \frac{\sin \alpha_1}{d_1}, \text{ and } \frac{\sin c_2}{a} = \frac{\sin \alpha_2}{d_2}$$

i.e.,

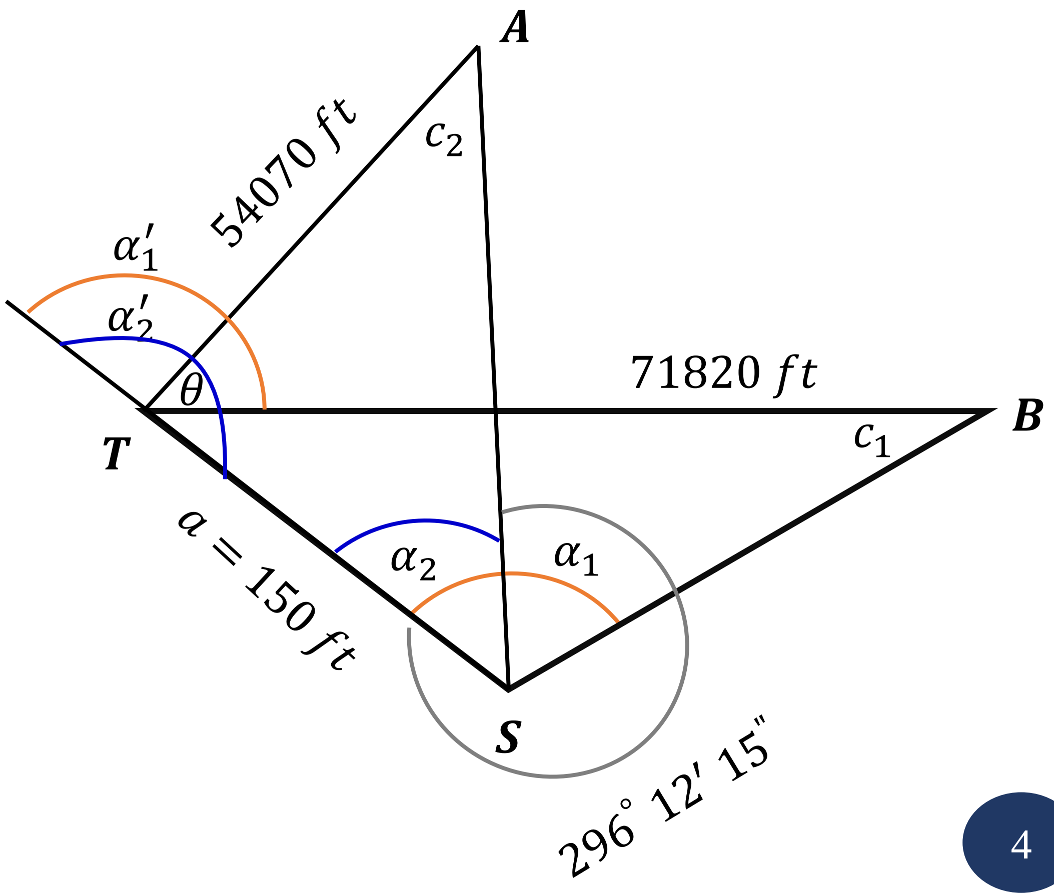
$$c_1 = sin^{-1} \frac{a \times \sin \alpha_1}{d_1 \times \sin 1''}$$

$$c_2 = sin^{-1} \frac{a \times \sin \alpha_2}{d_2 \times \sin 1''}$$

Such that c_1, and c_2 are given in seconds.

Numerical Exercises

- (1) Directions were observed from a satellite station S, 150 ft apart from the main triangulation station T such that direction SA = $00^{\circ}\ 00'\ 00''$, SB = $71^{\circ}\ 54'\ 30''$, and ST = $296^{\circ}\ 12'\ 15''$. Compute the subtended angle (ATB) at the main station T if the length of sides TA = 54070 ft, and TB = 71280 ft.



$$\alpha_2 = 360 - 296^{\circ}\ 12'\ 15'' = 63^{\circ}\ 47'\ 45''$$

$$\alpha_1 = \alpha_2 + \widehat{ASB} = 63^{\circ}\ 47'\ 45'' + 71^{\circ}\ 54'\ 30'' = 135^{\circ}\ 42'\ 15''$$

$$c_1 = sin^{-1}\frac{a \times \sin\alpha_1}{d_1 \times \sin 1''} = sin^{-1}\frac{150 \times \sin 135^{\circ}\ 42'\ 15''}{71280 \times \sin 1''} = 303''$$

$$c_2 = sin^{-1}\frac{a \times \sin\alpha_2}{d_2 \times \sin 1''} = sin^{-1}\frac{150 \times \sin 63^{\circ}\ 47'\ 45''}{54070 \times \sin 1''} = 513.41''$$

$$\theta = \widehat{ATB} = \alpha_1' - \alpha_2' = 71^{\circ}\ 50'\ 59.72''$$

- (2) Instead of a main triangulation station A which is not accessible, a theodolite was setup at station S 10.44 ft apart and approximately south-east from A. The observed direction to A was $43^{\circ}\,22'\,15''$ while that to B was $158^{\circ}\,48'\,57''$ and that to C was $227^{\circ}\,25'\,41''$. The lengths of AB and AC are 16560 ft and 21580 ft, respectively. Compute angle BAC.

$$\alpha_1 = directions(SB - SA) = 115^{\circ}\,26'\,42''$$

$$\alpha_2 = 360 - directions(SC - SA) = 175^{\circ}\,56'\,34''$$

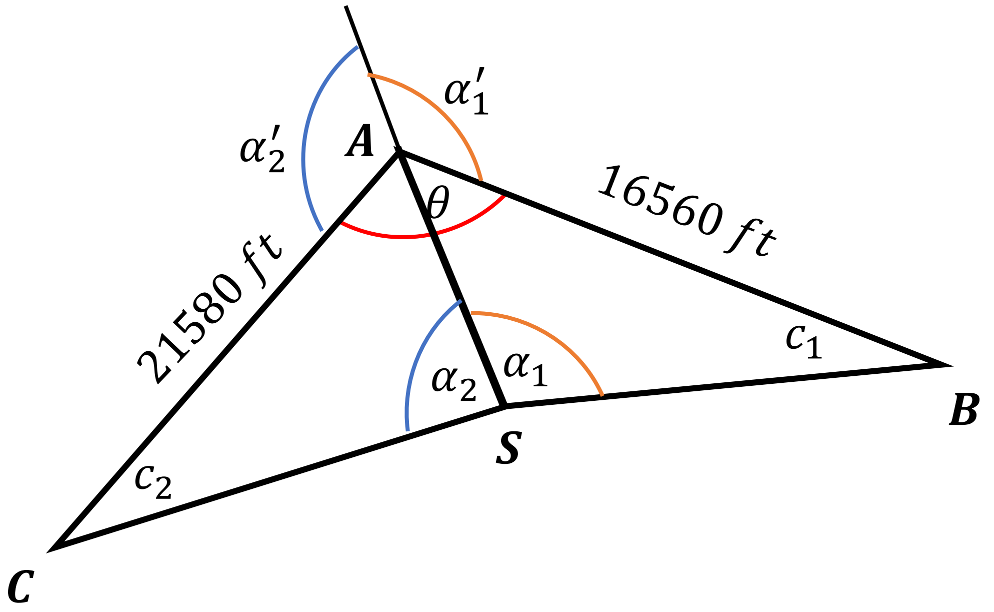
$$c_1 = sin^{-1}\frac{a\times\sin\alpha_1}{AB\times\sin 1''} = sin^{-1}\frac{150\times\sin 135^{\circ}\,42'\,15''}{71280\times\sin 1''} = 117.42''$$

$$c_2 = sin^{-1}\frac{a\times\sin\alpha_2}{AC\times\sin 1''} = sin^{-1}\frac{150\times\sin 63^{\circ}\,47'\,45''}{54070\times\sin 1''} = 7''$$

$$\alpha_1' = \alpha_1 + c_1$$

$$\alpha_2' = \alpha_2 + c_2$$

$$\theta = \widehat{BAC} = 360 - (\alpha_1' - \alpha_2') = 68^{\circ}\,34'\,39.58''$$



End of Presentation

GED
Geomatics Engineering Department

THANK YOU